

Quasi-stationary distributions for population processes

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Our aim in this course is to study the asymptotic behavior of the size of some isolated biological populations modelled by different random processes. Competition for limited resources impedes natural populations with no immigration to grow indefinitely, and they are all doomed to become extinct after some finite time. However, this extinction time can be large compared to human timescale and certain species may survive for long periods before extinction actually occurs. This transient behavior is captured by the mathematical notion of quasi-stationarity.

Specifically, a quasi-stationary distribution for a process Z is a probability measure m satisfying the following property. If the initial law of the process Z is m , then the conditional law of the process to be non extinct at some time t is equal to m , for every time t .

In this course, we will study the existence and uniqueness of quasi-stationary distributions for different models of populations in continuous time, taking into account competition between individuals, for small or large populations. Each case uses different probabilistic and analytic tools. We will study the possible long-time convergence of the law of the process conditioned on non extinction to a quasi-stationary distribution. Usually the existence of quasi-stationary distribution is not obtained by a constructive method. We will explain a particle method based on Fleming-Viot systems to construct an algorithm for their simulation.