The coalescent

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Lecture 1. Kingman's coalescent. This probabilistic model appeared in 1982. It is a very natural one. It plays a central role for the description of genealogies, very much like Brownian motion in finance, or the Poisson process in queuing theory. It is by now used for statistical sudies in population genetics, see O. François' lectures. We will show that the coalescent can be obtained as a limit model as the size of the population tends to infinity, describing the blood relationship among n individuals sampld in the population. We shall describe the coalescent. We will show in particular that Kingman's coalescent "comes down from infinity", and we will see at which speed.

Lecture 2. The Ewens ampling formula. We now consider the coalescent with mutations (in the so-called "infinite alleles model", where each new mutation gives birth to an individual of a completely new type), which appear according to a Poisson process of given intensity. The celebrated Ewens sampling formula, which dates back from 1972, descirbes the probability of finding in the sample a individuals of a first type, b individuals of a second type,...

Lecture 3. The Wright-Fisher diffusion. We discuss the evolution of the proportion of a given type in a population of fixed size, composed of individuals of two types. We show that the Wrigt-Fisher diffusion is obtained in the limit when the size of the population tends to infinity. We establish a duality relation between the Wright-Fisher diffusion and Kingman 's coalescent.

Lecture 4. The modified lookdown construction. We show that the Wright-Fisher diffusion can be obtaines as an a. s. limit if we use the so-called modified look-down cosntruction, which permits to consider directly a population of infinite size. We will recall de Finetti's theorem about exchangeable sequences of random variables.