Branching processes and splitting trees

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First lecture. We begin with (Markovian) branching processes in discrete time, called Bienaymé-Galton-Watson (BGW) processes. We review some basics like extinction-expansion dichotomy, one-dimensional marginals using probability generating functions, computation of extinction probability. Then, we study the asymptotic behaviour of branching processes, both unconditionally (Kesten-Stigum theorem) and conditionally on non-extinction (Yaglom quasistationary limit and Q-process).

Second lecture. Continuing with BGW trees, we give the spine decomposition of the tree conditioned to be never extinct: one infinite branch with independent finite trees growing off it.

We will also give a representation of the genealogy of an infinite population which is also doubly infinite in time. This allows us to find the joint law of the size of a quasi-stationary population and of the time to its most recent common ancestor. We introduce the coalescent point process for BGW trees.

Third lecture. We show how to code the genealogy of a BGW tree thanks to a killed random walk and the associated so-called height process. An application to the total progeny of the tree will be studied. Deeper understanding of the law of the total progeny will possibly be given thanks to Dwass' identity, via the ballot theorem.

We show other examples of branching processes with emphasis on modelling choices : Markov branching processes in continuous time, comprising in particular the linear birth-death process, Bellman-Harris processes, Crump-Mode-Jagers processes, Jirina processes.

Fourth lecture. We introduce splitting trees, whose width is a (generally not Markovian) branching process, in fact a binary, homogeneous Crump-Mode-Jagers process.

Proving that the contour process of splitting trees is a (killed) compensated compound Poisson process, we derive a number of properties of these trees including the coalescent point process. If time allows, we will consider the distribution of types in a population whose dynamics is that of splitting trees undergoing mutations at constant rate.