Remarks to Shoham's Temporal Logic

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Abstract

We describe a problem in Shoham's system of temporal logic and present a solution.

1 Introduction

1.1 General Remarks

This paper's intention is very limited : To point out a problem in Shoham's system of temporal logic and to offer a solution. In addition, both problem and solution are rather technical. Thus, the reader will find no general discussion of representation of time, neither in the text, nor in the bibliography, and will need a copy of [S] or [S0] at hand. For further reading, we suggest [S1] (with extended bibliography), [A], [AH].
We leave all motivation and comments beyond our modifications to Shoham's original articles.
Our reformulation of the boundary between syntax and semantics is largely a matter of taste, not central in any way, and we will not defend it beyond the following comment : In our example, we treat a predicate (or function) that is time-dependent. So, not to mention time-dependency at all on the syntactical level would seem to us somewhat cheating, since we have to interpret in a time-dependent way anyhow. After all, the language used should reflect some of the expressive power we intend to give it, and we see no point in hiding everything in the semantics.
The remarks in this paper apply to the system of temporal logic as presented in [S] and [S0]. Since [S] is the later and more elaborate article we refer to [S] only.
All undefined notation will be from [S].
We now examine Shoham's first order case and discuss a problem. In addition, we will set up a background scenario for illustration, which will be represented later on in the modified system of Shoham.

1.2 Problem and Scenario

The Problem :
Consider Shoham's President example. "President" is a non-temporal function symbol, $President \in F$. Let t be a temporal term and look at M_4 . $M_4(t, t, President)$ is then a function, that's o.k. so far. (We forget about $t = 10^6$ BC for the moment. We will solve that difficulty as a side effect later on.) But, look, what about t=1900, t'=1950 ?

Which function is $M_4(1900, 1950, President)$?
Obviously, none (in any reasonable sense), because the presidents have changed in the meantime!
Similar considerations apply, of course, to relations.
The scenario :
We want to express "from 1900 to 1950 the president of the US has been a Republican" (we do not bother about historical correctness). We might have in mind :
1) The (predicate) Republican party has changed, some people joined, others left, or died.
2) The presidents of the US have changed, too, so the function "president" has changed.
3) But : Over all that time, whoever was president, was at that time Republican.
We might express 3) also as
3') 1900-1920 Hoover was president, he was a Republican, 1920-1940 Roosevelt was president, he was a Republican, 1940-1950 Eisenhower was president, he was a Republican.
Note, however, that 3') might easily be an infinite set of sentences, if we look at continous developments like the movement of a body in space.

To put it graphically :

1.3 Technical Remarks

Shoham (personal communication) has pointed out to the author that, in the original paper, the correctness of the crucial lemma 2 was based on nothing but the author's intuition. This lemma, and its counterparts for relations and the $\exists$-quantifier, are central for anything sensible : we have to be independent of the specific choice of subintervals. The reader may convince himself by either looking at our scenario, or at the theory of integration, which may serve as a good model. To obtain our lemma, we are forced to accept something like liquidity (see [S]) for the predicates. (We have called liquidity "Time Decency" in a second version of the paper and are grateful to the referees for pointing out to us that both mean the same.) If we want to do without full liquidity, the author sees no way but to largely increase the basic formalism, e.g. carrying around for each predicate its own little algebra of admissible time intervals. We do not consider this a good solution (just think of complex formulae!). Instead, we suggest that further research on temporal representation should try to work with simple basic structures and put any complications into the axiom sets. So, for simplicity, *we consider here only liquid predicates.* And these pages should be read as a research note offering a partial solution rather than as an effort to firmly establish any system of temporal reasoning.

Our central modification of Shoham's system are in the definitions of FN and RL (partial functions and dependency on time), and in the "refinement"-cases in defining the interpretation of functions, relations, and the existential quantifier for non-temporal variables.

2 An (Obvious) Solution

2.1 Outline

We now proceed to modify Shoham's development of the first order case.
The problem was that functions and relations change over time, so they need not be constant over every interval. Thus, they need not be defined if we consider arbitrary intervals. Yet, we want to preserve the intervals as the basic time structure (and not go back to time-points, which would solve our problem immediately, but not the one with the president 10^6 BC !). So we just make time dependence and partial definitions explicit. For that purpose, we append to any non-temporal function and relation symbol a time interval. For simplicity, we assume that the time intervals coincide up to the level of atomic formulae. Time-invariant expressions like constants will be considered to have any time interval appended (we might think of $(-\infty, \infty)$, if we like).
Remarks :
1) We could elaborate, by taking e.g. the intersection of the time-intervals involved in atomic formulae. But, what if the intersection is empty? Think of sentences like "the king of France in 1987". As we don't want to go into

philosophical considerations, we ban these problems from the syntactical level to the semantical one, and let the outside world decide.
2) We stay with Shoham in assigning temporal expressions constant values over time. We keep in mind, however, that this keeps us from the straightforward expression of things like "now", "today" etc.
3) To have a clearer boundary between syntax and semantics, we change the interpretation of time points a little bit.

In our definitions, we always assume that the arity of functions etc. fits. Partly, the definitions are as in [S] or [S0], for the sake of readibility, we will give the full picture.

2.2 Syntax

The alphabet (like in [S]) : TC, a set of time point symbols; C, a set of constant symbols, disjoint from TC; TV, a set of temporal variables; V, a set of non-temporal variables, disjoint from TV; TF, a set of temporal function symbols like addition (Remark : We leave aside the question that it is unclear what 1.1.1980+1.1.1981 is supposed to be); F, a set of non-temporal function symbols, disjoint from TF; R, a set of non-temporal relation symbols.
The temporal terms (essentially like in [S]) : (1) all elements of TC are temporal terms (2) all elements of TV are temporal terms (3) if $t_1 \cdots t_n$ are temporal terms, f is in TF, then $f(t_1 \cdots t_n)$ is a temporal term.
In all cases, we say that temporal terms have any time-interval (see above).
Non-temporal terms : (1) all elements of C are non-temporal terms (with any time-interval) (2) all elements of V are non-temporal terms (with any time-interval) (3) if $t_1 \cdots t_n$ are non-temporal terms with the same time-interval $< \tau, \tau' >$, τ, τ' are temporal terms, f is in F, then $f_{<\tau,\tau'>}(t_1..t_n)$ is a non-temporal term with the time interval $< \tau, \tau' >$.
Comment:
Consider our scenario. Suppose f is the function $President : Countries \rightarrow People$, $\tau = 1900, \tau' = 1920, t_1 = US$. So, $f_{<\tau,\tau'>}(t_1)$ should be read as : the president of the US from 1900 to 1920 (i.e. Hoover). If, however, $\tau' = 1950$, "the president of the US from 1900 to 1950" (as a person) is syntactically correct, but (semantically) undefined, as there is no such person. Indeed, it is the knowledge about the actual state of the world, which makes this undefined. So, this will be reflected by semantics, and the function "President" will not be defined for all time-intervals and countries : it will be a partial function.
Atomic temporal wffs : if t,t' are temporal terms, then t=t', $t \prec t'$ are temporal wffs, with any time interval.
Atomic non-temporal wffs : if $t_1 \cdots t_n$ are non-temporal terms with same time interval $< \tau, \tau' >$ for all t_i, $r \in R$, then $r_{<\tau,\tau'>}(t_1..t_n)$ is a non-temporal wff, with the time interval $< \tau, \tau' >$.
Comment :
Analogous to functions, the truth-value of predicates will depend on the time-

intervals considered. Extending our scenario to $< 1900, 1980 >$ will give an example.
Non-atomic wffs : if ϕ, ψ are wffs, then $\phi \wedge \psi$, $\neg\phi$ are wffs; if $\phi(z)$ is a wff, then $\forall z \phi(z)$ is a wff etc.
Remark : We follow [S] in not developing any proof theory, which should be obvious once we have fixed semantics and our general background of how to deal with "the king of France in 1987" etc.

2.3 Semantics

An interpretation is an n-tupel
$J =< TW, \leq, W, TFN, FN, RL, M =< M_1, M_2, M_3, M_4, M_5 >>$. (In the first part are the things we are talking about, and the M_i 's do the assignment.)
TW : a non-empty set of time points, $\leq$: a reflexive total order on TW, I : the set of intervals over TW, just for abbreviation, W : a non-empty universe of individuals, disjoint from TW, TFN : a set of functions $f : TW^{n_f} \rightarrow TW$, n_f the arity of f, FN : a set of partial functions $f : I \times W^{n_f} \rightarrow W$, n_f, again the arity, RL : a set of relations $r \subseteq I \times W^{n_r}$, n_r the arity of r.
(See the section on syntax for comments.)
We now formulate our "liquidity"-requirements. First, we need the fact that something is constant over $< \tau, \tau' >$ iff it is so over any refinement $\tau = \tau_0 < \cdots < \tau_\alpha = \tau'$, (and then has the same value). We make this explicit. For $f \in FN$ and $\tau = \tau_0 \leq \cdots \leq \tau_\alpha = \tau'$ we require $f(< \tau, \tau' >, t_1, .., t_n)$ is defined iff $f(< \tau_\beta, \tau_{\beta+1} >, t_1, .., t_n)$ are all defined for $0 \leq \beta < \beta + 1 \leq \alpha$ and equal and then $f(< \tau, \tau' >, t_1, \cdots, t_n) = f(< \tau_\beta, \tau_{\beta+1} >, t_1, \cdots, t_n)$ for all β . Second, for $r \in RL$ and $\tau = \tau_0 \leq \cdots \leq \tau_\alpha = \tau'$ we require $r(< \tau, \tau' >, t_1, \cdots, t_n)$ iff $r(< \tau_\beta, \tau_{\beta+1} >, t_1, \cdots, t_n)$ for all $0 \leq \beta < \beta + 1 \leq \alpha$

The functions : $M_1 : TC \rightarrow TW$, $M_2 : C \rightarrow W$, $M_3 : TF \rightarrow TFN$, $M_4 : F \rightarrow FN$, $M_5 : R \rightarrow RL$

A variable assignment is a pair $VA =< VAT, VAV >$ of functions $VAT : TV \rightarrow TW$, $VAV : V \rightarrow W$.

We now define the interpretation $MVA =< M, VA >$ by induction :
The meaning of temporal terms : $MVA(v) := VAT(v)$ for $v \in TV$, $MVA(c) := M_1(c)$ for $c \in TC$, $MVA(f(t_1 \cdots t_n)) := M_3(f)(MVA(t_1) \cdots MVA(t_n))$ for $f \in TF$, t_i temporal terms.
The meaning of non-temporal terms :
$MVA(v) := VAV(v)$ for $v \in V$, $MVA(c) := M_2(c)$ for $c \in C$.
Let $f \in F$, t_i be non-temporal terms, τ, τ' temporal terms.
We want to define $MVA(f_{<\tau,\tau'>}(t_1 \cdots t_n))$. Case 1 : $MVA(t_i)$ is defined, and $M_4(f)(< MVA(\tau), MVA(\tau') >, MVA(t_1) \cdots MVA(t_n))$ too. Then let this be the value. Case 2 : otherwise, but there is a monotonic sequence $< \tau_i : i \leq \alpha >, \tau_0 = \tau, \tau_\alpha = \tau'$, such that $MVA(t_i[< \tau, \tau' > / < \tau_\beta, \tau_{\beta+1} >])$ (this is of course the result of replacing τ by τ_β etc.) is defined for all $\beta < \alpha$, and $M_4(f)(< MVA(\tau_\beta), MVA(\tau_{\beta+1}) >, MVA(t[< \tau, \tau' > / < \tau_\beta, \tau_{\beta+1} >]) \cdots)$ is

defined and equal for all $\beta < \alpha$. Then, take any such $< \tau_i : i \leq \alpha >$, and let the above be the value.
Case 3 : none of the above Leave the function undefined for that interval
$< MVA(\tau), MVA(\tau') >$
Remark : In case 2, we express the fact that f need not be injective, this might compensate a change in value of the $\tau_i' s$.
Motivation : Instead of considering the predicate "Republican" of our scenario, you may think here of the function $people \rightarrow party$ and the term
$party(President_{<1900,1950>}(US))$.
For details, see the section "Scenario Reconsidered".
We now show that in Case 2, the value is independent of the choice of the $< \tau_i : i \leq \alpha >$. The idea is, of course, to consider common refinements of the intervals. The credulous reader may skip the following two lemmas.

Lemma 1 *Let $\tau < \tau' < \tau"$, t a term. Then $MVA(t_{<\tau,\tau">})$ is defined iff $MVA(t_{<\tau,\tau'>})$ and $MVA(t_{<\tau',\tau">})$ are defined and then*
$MVA(t_{<\tau,\tau">}) = MVA(t_{<\tau,\tau'>}) = MVA(t_{<\tau',\tau">})$.

Proof : (Induction on the complexity of t)
For $t = v \in V$, $t = c \in C$, this is trivial. Let $t = f(t_1 \cdots t_n)$ and assume the lemma to be true for t_i .
a) Suppose $MVA(t_{<\tau,\tau">})$ is defined by case 1. So $MVA(t_{i,<\tau,\tau">})$ is defined.
By induction hypothesis, $MVA(t_{i,<\tau,\tau'>})$ and $MVA(t_{i,<\tau',\tau">})$ are defined and $MVA(t_{i,<\tau,\tau">}) = MVA(t_{i,<\tau,\tau'>}) = MVA(t_{i,<\tau',\tau">})$. So by liquidity of FN,
$M_4(f)(< MVA(\tau), MVA(\tau") >, t_{1,<\tau,\tau">} \cdots t_{n,<\tau,\tau">}) =$
$M_4(f)(< MVA(\tau), MVA(\tau') >, t_{1,<\tau,\tau'>} \cdots t_{n,<\tau,\tau'>}) =$
$M_4(f)(< MVA(\tau'), MVA(\tau") >, t_{1,<\tau',\tau">} \cdots t_{n,<\tau',\tau">})$.
Suppose $MVA(t_{<\tau,\tau'>})$ and $MVA(t_{<\tau',\tau">})$ are defined.
Arguing as above, only "upward", $MVA(t_{i,<\tau,\tau">})$ are all defined and equal to $MVA(t_{i,<\tau,\tau'>})$ etc.
So we apply liquidity again and conclude that $MVA(t_{<\tau,\tau">})$ is defined and
$MVA(t_{<\tau,\tau">}) = MVA(t_{<\tau,\tau'>}) = MVA(t_{<\tau',\tau">})$.
b) Suppose $MVA(t_{<\tau,\tau">})$ is defined by case 2, $\tau = \tau_0 \leq \cdots \leq \tau_\alpha = \tau"$.
Let $\tau_\beta \leq \tau' \leq \tau_{\beta+1}$. Applying a) to $\tau_\beta, \tau_{\beta+1}, \tau'$ and liquidity again will show the desired result.

Lemma 2 *The definition in Case 2 is independent of the chosen sequence*
$< \tau_i : i \leq \alpha >$.

Proof : (Sketch)
Let $< \tau_i : i \leq \alpha >$ and $< \sigma_i : i \leq \beta >$ be given. Take the common refinement $< \mu_i : i \leq \gamma >$ (i.e. all τ_i ,σ_i are some μ_j). Applying Lemma 1 and liquidity will give the proof.
Let S = $< J, VA >$
Validity of atomic temporal wffs (like in [S]) :

$S \models t = t'[VA]$ iff MVA(t)=MVA(t'),
$S \models t \prec t'[VA]$ iff $MVA(t) < MVA(t')$
Validity of atomic non-temporal wffs : This is defined just like the meaning of non-temporal terms :
$S \models r_{<\tau,\tau'>}(t_1 \cdots t_n)$ iff
Case 1 :
$MVA(t_i)$ is defined, and $(< MVA(\tau), MVA(\tau') >, MVA(t_1) \cdots MVA(t_n)) \in M_5(r)$
Case 2 : otherwise, but there is a monotonic sequence $< \tau_i : i \leq \alpha >, \tau_0 = \tau, \tau_\alpha = \tau'$, such that $MVA(t_i[< \tau, \tau' > / < \tau_\beta, \tau_{\beta+1} >])$ is defined for all $\beta < \alpha$, and for all $\beta < \alpha$
$(< MVA(\tau_\beta), MVA(\tau_{\beta+1}) >, MVA(t_1[< \tau, \tau' > / < \tau_\beta, \tau_{\beta+1} >]) \cdots$
$MVA(t_n[< \tau, \tau' > / < \tau_\beta, \tau_{\beta+1} >]) \in M_5(r)$.

Independence of the specific τ_i is just as with functions.

Non-atomic wffs : $S \models \phi \wedge \psi[VA]$ iff $S \models \phi[VA]$ and $S \models \psi[VA]$; $S \models \neg\phi[VA]$ iff $not(S \models \phi[VA])$;
$S \models \exists x \phi(x)$
Case I : x is a temporal variable : Then $S \models \exists x \phi(x)$ iff there is a $\tau \in TW$ such that $S \models \phi(\tau)$.
Case II : x is a non-temporal variable : Let $< \tau_i : i \leq \gamma >$ be the enumeration of all time points occurring in ϕ in increasing order. Then $S \models \exists x \phi(x)$ iff there is a refinement $< \tau_i : i \leq \alpha >$ of $< \tau_i : i \leq \gamma >$ and for each $< \tau_\beta, \tau_{\beta+1} >$ there is a $d_\beta \in W$, such that $S \models \phi_{<\tau_\beta, \tau_{\beta+1}>}[d_\beta]$.
To show independence of the specific $< \tau_i : i \leq \alpha >$ is again along the lines of Lemma 1 and 2.
Remark : We might be tempted to get rid of the sequences $< \tau_i : i \leq \alpha >$ by a recursive definition like $r_{<\tau,\tau'>}(t)$ iff for all $\tau" \in [\tau, \tau']$ $r_{<\tau,\tau">}(t)$ and $r_{<\tau",\tau'>}(t)$. But we do not know that there are no infinite descending chains of intervals, so our recursive definition need not be well-founded.

2.4 Scenario Reconsidered

It is now a triviality to express our scenario :
$Republican_{<1900,1950>}(President_{<1900,1950>}(US))$ iff
$Republican_{<1900,1920>}(President_{<1900,1920>}(US))$ and
$Republican_{<1920,1940>}(President_{<1920,1940>}(US))$ and
$Republican_{<1940,1950>}(President_{<1940,1950>}(US))$ iff
$Republican_{<1900,1920>}(Hoover)$ and $Republican_{<1920,1940>}(Roosevelt)$ and
$Republican_{<1940,1950>}(Eisenhower)$ iff True.

3 Conclusion

We have given a scenario that can't be translated into Shoham's original system of temporal logic, at least not straightforwardly, due to the fact that the values of functions and relations may change over time. We have modified Shoham's system (for liquid predicates) to make our scenario easily and according to intuition expressible.

References

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