SEMANTICS FOR DEFEASIBLE INHERITANCE

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Abstract

We discuss a (class of) semantics for defeasible inheritance, based on "normal" subsets.

1 Introduction

We will propose a semantics for non-monotonic inheritance which can handle preclusion. Our approach is based on formalizing the notion of a "normal" subset, allowing us to state e.g. "normally, all p are q". Since for preclusion, direct links are in a stronger way true than valid paths, we express this by different degrees of "normality", resulting in a many-valued semantics. Primarily, our semantics is intended for the directly sceptical approach; for extensions, we suggest a combination with possible worlds. Space does not permit us to discuss defeasible inheritance in general, the reader is referred to [T], [HTT], [THT], [MS].

All diagrams are collected in an appendix and referred to as Ex .... .

2 The Semantics

Outline We formalize propositions like "ϕ is normally valid in c" by defining sets of "normal" (large, containing the "normal" or "important") subsets, called $\mathcal{N}-systems$ over c, and use this technique to construct models for (almost) any notion of validity in nets. (Though, in our opinion, the extensions approach is best handled by a combination of these semantics and a possible world technique.) For more details and motivation, see the Discussion following Proposition 2.3. The first construction has one defect : we build models from the full theory, not from the axioms. To do the latter, we have to refine the method (starting in Definition 2.3) to accomodate preclusion. The basic reason is, that direct links are for preclusion in a stronger sense true than valid paths. We thus define $\mathcal{N} - families$ (decreasing sequences of $\mathcal{N} - systems$) which give increasingly strong notions of truth and use them to construct models suitable to express preclusion. The concept of $\mathcal{N} - families$ is general enough to permit the construction of semantics for more complicated notions of validity in nets, like those suggested to solve the Double Diamond problem in [S1]. Moreover, since these lines have first been written, we used $\mathcal{N} - systems$ to give normal open defaults a semantics. Defaults are read as generalized quantifiers, added to predicate logic, interpreted by such "large" subsets of the universe. Thus, we can go beyond usual default theories, e.g. negated and iterated normal defaults will have a precise meaning. See [S2] for details and [L] for an implementation.

Definition 2.1 *Call* $\mathcal{N}(c) \subseteq \mathcal{P}(c)$ *($\mathcal{P}$ the Power Set Operator) a* $\mathcal{N} - system$ *over c iff*

a. $c \epsilon \mathcal{N}(c)$

b. $a \epsilon \mathcal{N}(c)$, $a \subseteq b \subseteq c \rightarrow b \epsilon \mathcal{N}(c)$

c. $a, b \epsilon \mathcal{N}(c) \rightarrow a \cap b \neq \emptyset$ *if* $c \neq \emptyset$ *(thus,*

$\emptyset \not\in \mathcal{N}(c)$)

Remark 2.1 *a) $\mathcal{N}$ stands for normal. We formalize "ϕ is normally valid in c" by $\exists a \epsilon \mathcal{N}(c). \forall x \epsilon a. \phi(x)$.*
b) There is nothing to prevent e.g. $\mathcal{N}(c) = \{a \subseteq c : x \epsilon a\}$ for some fixed $x \epsilon c$. This might seem pathological. But, this x might be a very prototypical case, and thus have intuitive meaning.

Lemma 2.2 *Let $\mathcal{L} \subseteq \mathcal{P}(X)$ be such that $A, B \epsilon \mathcal{L} \rightarrow A \cap B \neq \emptyset$. Then $\mathcal{N}(X) := \{A \subseteq X : \exists B \epsilon \mathcal{L}. B \subseteq A\} \cup \{X\}$ is a $\mathcal{N}-system$ over X.*

Example 2.1 *Let α be any ordinal, and $U := \{f : \alpha \rightarrow 2 = \{0,1\}\}$. For $i < \alpha$ let $X_i := \{f \epsilon U : f(i) = 1\}$, $X"_i := \{f \epsilon U : f(i) = 0\} = U - X_i$. For $j < \alpha$ let $I_j \subseteq \alpha$, $I"_j \subseteq \alpha - \{j\}$ such that $I_j \cap I"_j = \emptyset$, and let $\mathcal{L}_j := \{X_j \cap X_i : i \epsilon I_j\} \cup \{X_j \cap X"_i : i \epsilon I"_j\}$. Then $\mathcal{L}_j$ sat. the prerequisites of Lemma 2 for X_j and all j. Consequently, $\mathcal{L}_j$ so defined generates a $\mathcal{N}-system$ over X_j for all $j < \alpha$.*

Definition 2.2 *Let Γ be a net, $V(\Gamma)$ its vertices. We call $V(\Gamma)$ the language of Γ. A $\Gamma-structure$ Σ is a tripel consisting of a universe U, subsets of the universe, [p], for the $p \epsilon V(\Gamma)$, and $\mathcal{N}-systems$ over these subsets :*

$\Sigma = < U , \{[p] : p \epsilon V(\Gamma)\} , \{\mathcal{N}(p) : p \epsilon V(\Gamma), \mathcal{N}(p)$ *a* $\mathcal{N}-system$ *over* $[p]\} >$.

We define validity : $\Sigma \Vert\!\models p \rightarrow q$ iff $[p] \cap [q] \epsilon \mathcal{N}(p)$. $\Sigma \Vert\!\models p \not\rightarrow q$ iff $[p] - [q] \epsilon \mathcal{N}(p)$.

and say Σ is a $\models -model$ of Γ (for $\models$ a notion of validity in nets), iff for all p,q ($\Sigma \Vert\!\models p \rightarrow q$ iff $\Gamma \models p \rightarrow q$) and ($\Sigma \Vert\!\models p \not\rightarrow q$ iff $\Gamma \models p \not\rightarrow q$) .

Proposition 2.3 *Let Γ be a net and $\models$ be any notion of validity of links, which 1. does not permit $\Gamma \models p \rightarrow q$ and $\Gamma \models p \not\rightarrow q$ at the same time (note the subsequent Discussion 2., however) 2. satisfies $\Gamma \models p \rightarrow p$ for all p. Then there is a (canonical) $\models -model$ Σ for Γ.* □

Outline of Proof : Let $U := \{f : V(\Gamma) \rightarrow 2\}$, $X_p := \{f \epsilon U : f(p) = 1\}$, $X"_p := \{f \epsilon U : f(p) = 0\} = U - X_p$, $I_p := \{q : \Gamma \models p \rightarrow q\}$, $I"_p := \{q : \Gamma \models p \not\rightarrow q\}$, $\mathcal{L}_p := \{X_p \cap X_q : q \epsilon I_p\} \cup \{X_p \cap X"_q : q \epsilon I"_p\}$, $\mathcal{N}(p) := \{Y \subseteq X_p : \exists Z \epsilon \mathcal{L}_p. Z \subseteq Y\} \cup \{X_p\}$. Then $\Sigma := <$ U , $\{X_p : p \epsilon V(\Gamma)\}$, $\{\mathcal{N}(p) : p \epsilon V(\Gamma)\} >$ is a $\models -model$ of Γ. The proof in the non-trivial direction proceeds through an examination of the possible cases in the definition of $\mathcal{L}_p$ and $\mathcal{N}(p)$. Choosing suitable functions f will give the desired results. □

Discussion : 1. We first tried simpler approaches like subsets of the real plane and measures. But look at Ex1. Following the simpler interpretation, we have to read : Most of a_1 is in c, most of a_2 is in a_1, most of a_2 is not in c. So, a_2 has to be a lot smaller than a_1 is etc. In the end, some a_i is very much smaller than b_1 (or some b_i than a_1). But, then most of b_1 is in the complement of a_i, and we would have a negative arrow from b_1 to a_i, which simply is not there. If we took a lot of different models and intersections of true statements, we could easily run into situations like Ex2, and be unable to fully model a sceptical approach. So, taking different measures ($\mathcal{N}(p)$) for different p's in $V(\Gamma)$ seems necessary.

2. To represent directly contradictory information like the Garbage In Rule of Thomason, we can construct 2 models in the above way, much as his construction for the 4-valued monotonic case - see [THT2].

3. We have a 3-valued semantics : $[p] \cap [q] \epsilon \mathcal{N}(p)$, $[p] - [q] \epsilon \mathcal{N}(p)$ and neither. We would like to emphasize very strongly that the latter case is not intended to express lack of information : There simply is no decision wrt. q possible ! Otherwise, we might speculate, if $p \rightarrow q$, then ...., running into differences

between direct scepticism and $\bigcap Extensions$, as exemplified in Ex2, and discussed in more detail in [S3]. (We show there that under very weak assumptions, no directly sceptical approach with finitely many truth values can match exactly the intersection of extensions.) The $\bigcap Ext$ approach might be better modelled by a combination with a possible worlds semantics, where decisions in Nixon-Diamond's are modelled by a branching point.

4. Looking back to 1., we can see that the diagram of Ex2 simply makes no sense as a description of a simple situation like the above mentioned common measure one. In this way, a clear semantics is more than a formally satisfying enterprise. We have shown how to give some sense to a very broad class of diagrams and definitions of validity. Inversely, using more restrictive semantics might give good characterizations of classes of simpler diagrams and notions of validity.

5. It should be easy to incorporate relations, functions etc. into our semantics, too. By the way, strict inheritance becomes the simple restriction $\mathcal{N}(c) = \{c\}$, so there is no problem here with representing strict and defeasible inheritance.

6. Our next task will be to construct a model from the axioms, not from the full theory. The lack of stability in all approaches that work with direct links in the case of preclusion necessitates a more fine-grained semantics if we want to express reasoning, not only truth, in nets : $p \to q$ as a direct link is in a stronger sense true than $p \to q$ by $p \to r \to q$. In our examination of the Double Diamond-problem (see [S1]), we have suggested still more discriminating formalisms, so we immediately prepare a general solution. The development is very parallel to the one already done above.

Definition 2.3 *Let γ be any ordinal. Call $< \mathcal{N}_i(c) : i < \gamma >$, $\mathcal{N}_i(c) \subseteq \mathcal{P}(c)$ a $\mathcal{N}-family$ over c iff*

a. $c\epsilon\mathcal{N}_i(c)$ for all i

b. $a\epsilon\mathcal{N}_i(c)$, $a \subseteq b \subseteq c \to b\epsilon\mathcal{N}_i(c)$ for all $i < \gamma$

c. $< \mathcal{N}_i(c) : i < \gamma >$ is decreasing, i.e. $a\epsilon\mathcal{N}_i(c), j < i \to a\epsilon\mathcal{N}_j(c)$

d. $a\epsilon\mathcal{N}_i(c)$, $b\epsilon\mathcal{N}_j(c) \to a \cap b \neq \emptyset$ for all i,j , if $c \neq \emptyset$.

Remark 2.4 *We have formalized γ degrees of normality. Condition d. says, that all $\mathcal{N}_i(c)$ are $\mathcal{N}-systems$ over c.*

Lemma 2.5 *Let $< \mathcal{L}_i : i < \gamma >$, $\mathcal{L}_i \subseteq \mathcal{P}(X)$ be such that $A, B\epsilon \bigcup_{k<\gamma} \mathcal{L}_k \to A \cap B \neq \emptyset$. Then $\mathcal{N}_i(X) := \{A \subseteq X : \exists B\epsilon \bigcup_{i \leq j < \gamma} \mathcal{L}_j . B \subseteq A\} \cup \{X\}$ for $i < \gamma$ is a $\mathcal{N}-family$ over X.*

Example 2.2 *Let α be any ordinal, $U := \{f : \alpha \to 2\}$. For $i < \alpha$ let $X_i := \{f\epsilon U : f(i) = 1\}$, $X"_i := \{f\epsilon U : f(i) = 0\} = U - X_i$. For $j < \alpha$, $i < \gamma$ let $I_{j,i}, I"_{j,i} \subseteq \alpha$, $j \not\epsilon I"_{j,i}$, $(\bigcup_{i<\gamma} I_{j,i}) \cap (\bigcup_{i<\gamma} I"_{j,i}) = \emptyset$. Let $\mathcal{L}_{j,i} := \{X_j \cap X_k : k\epsilon I_{j,i}\} \cup \{X_j \cap X"_k : k\epsilon I"_{j,i}\}$.*

Then $< \mathcal{L}_{j,i} : i < \gamma >$ satisfies the prerequisites of Lemma 5 for all X_j, $j < \alpha$. Thus, for all j, $\mathcal{N}_{j,i}(X_j) := \{A \subseteq X_j : \exists B\epsilon \bigcup_{i \leq k < \gamma} \mathcal{L}_{j,k} . B \subseteq A\} \cup \{X_j\}$ is a $\mathcal{N}-family$ of length γ over X_j. $\Box$

Definition 2.4 *A $\Gamma, \gamma - structure$ Σ is a tripel, consisting of a universe, subsets of the universe for all $p\epsilon V(\Gamma)$, and a $\mathcal{N}-family$ of length γ over these subsets :*

$\Sigma = < U , \{[p] : p\epsilon V(\Gamma)\} , \{< \mathcal{N}_i(p) : i < \gamma > : p\epsilon V(\Gamma), < \mathcal{N}_i(p) : i < \gamma >$ a $\mathcal{N}-family$ over $[p]\} >$.

We define γ degrees of validity : Let $i < \gamma$, then $\Sigma \Vdash\!\models_i p \to q$ iff $[p] \cap [q] \epsilon \mathcal{N}_i(p)$ and $\Sigma \Vdash\!\models_i p \not\to q$ iff $[p] - [q] \epsilon \mathcal{N}_i(p)$

and say for a notion $\models = < \models_i : i < \gamma >$ of validity in nets of γ degrees that Σ is a $\models -model$ of Γ iff Σ is a $\Gamma, \gamma - structure$ and for all $p, q\epsilon V(\Gamma)$, $i < \gamma$ $\Sigma \Vdash\!\models_i p \to q$ iff

$\Gamma \models_i p \to q$ *and* $\Sigma \Vvdash_i p \not\to q$ *iff* $\Gamma \models_i p \not\to q$.

Proposition 2.6 *Let* Γ *be a net and* $\models = <\models_i : i < \gamma >$ *a notion of validity of links, which 1. does not permit* $\Gamma \models_i p \to q$, $\Gamma \models_j p \not\to q$ *for any p,q,i,j at the same time 2. satisfies* $\Gamma \models_i p \to p$ *for all p,i 3. is increasing, i.e. satisfies* $\Gamma \models_i p \to q \Rightarrow \Gamma \models_j p \to q$ *for all p,q,* $i > j$ *(likewise for negative links) Then there is a (canonical)* $\models -model$ *for* Γ. □

(The proof closely parallels that of Proposition 2.3.)

We now have the tools to handle preclusion, again with the proviso that no $p \to q$, $p \not\to q \epsilon \Gamma$ simultaneously. But this is an inessential restriction, as we can - as hinted at above - develop two models in tandem. We extend $\Vvdash_i$ to handle paths too. This, along with the notion of a preclusion-structure, is made precise in

Definition 2.5 *Set w:=0, s:=1 (w for weak, s for strong),* $\gamma := 2 = \{w, s\}$. *Let* Σ *be a* $\Gamma, 2-$ *structure. We say* Σ *is a* $\Gamma, 2, p-structure$*, (p for preclusion), iff it satisfies*

a. $\Sigma \Vvdash_s p \to q \Rightarrow \Sigma \Vvdash_w p \to q$

b. $\Sigma \Vvdash_w \sigma \Rightarrow \Sigma \Vvdash_w \sigma^+$*, where* σ^+ *is the "contraction" of* σ *to its resulting proposition. c.* $\Sigma \Vvdash_w \sigma$, $\sigma = x_1 \ldots x_n z$ *iff 0.* $\Sigma \Vvdash_w x_1 \ldots x_n$, $\Sigma \Vvdash_s x_n z$ *1. For no* $i < n$, $\Sigma \Vvdash_s x_i z^-$ *(* $^-$ *stands for the negative case), and there is no* $\tau = x_1 \ldots y_1 \ldots y_m \ldots x_i$, $\Sigma \Vvdash_w \tau$ *and* $\Sigma \Vvdash_s y_k z^-$ *2. If there is* $\tau = x_1 \ldots y$, $\Sigma \Vvdash_w \tau$, $\Sigma \Vvdash_s yz^-$*, then* $x_1 \ldots yz^-$ *is precluded (here, we mean of course the semantical counterpart, i.e. the negation of 1.) (The negative arrows are handled entirely symmetrically.)*

Being a verbatim semantic counterpart of the following definition, it is trivial to see that a $\Gamma, 2, p-structure$ such that $\Sigma \Vvdash_s p \to q$ iff $p \to q \epsilon \Gamma$ is a correct model of the sceptical notion of inheritance :

Definition 2.6 *We build the set of valid paths,* $V = \bigcup_{m \epsilon \omega} V_m$ *inductively, by degree :*

Set $V_0 := \Gamma$.

Let $W := \bigcup_{m' < m} V_{m'}$ *be defined and consider* $\sigma := x_1..x_n z, deg(\sigma) = m$.

Then $\sigma \epsilon V_m$ *iff*

(0) $x_1 \ldots x_n \ \epsilon \ W$, $x_n z \ \epsilon \ \Gamma$

(1) σ *is not precluded in W*

(2) Every contradiction to σ *in W is precluded in W.*

Finally, set $\Gamma \models \sigma$ *iff* $\sigma \epsilon V$.

(It may suffice here to thing of deg as providing some suitable order for the inductive definition.)

3 Conclusion

We have constructed a 5-valued model which gives $\Sigma \Vvdash_w p \to q$ iff $\Gamma \models \sigma : p \to q$, σ not a direct link, and $\Sigma \Vvdash_s p \to q$ iff $p \to q \epsilon \Gamma$. So, strong validity is direct links, weak validity is valid paths (resp. their result), this distinction makes it possible to handle preclusion. A critique : We have offered no semantics, which can discriminate between different formalisms of inheritance, only one that gives them meaning. We have tried to investigate discriminating semantics, but so far without definite conclusions.

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