A CP Approach to the Balanced Academic Curriculum Problem

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23 September 2007

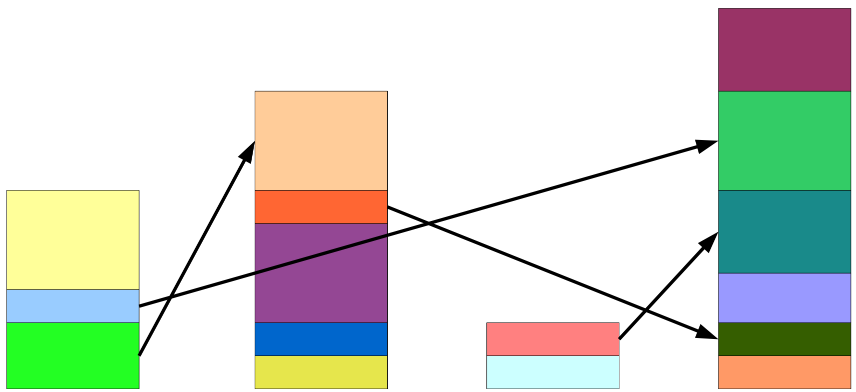
SymCon'07

The Balanced Academic Curriculum Problem

Given

- a set of courses (each course has a weight: number of credits).
- a set of periods.
- a set of prerequisite relationships among courses.

Find an assignation of courses to periods such that the prerequisites are respected and the academic load of each period is balanced.



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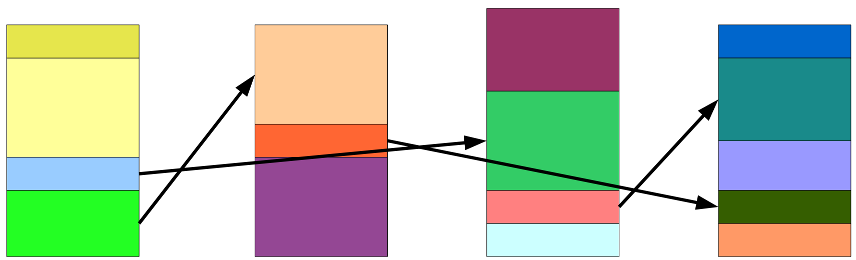


State-of-the-Art

- Castro and Manzano, 2001 : Branching heuristic
- Hnich et al., 2002 : CP and IP Models study
- Hnich et al., 2004 : CP/ILP approach
- Lambert et al, 2006 : CP/GA approach

Test Set : 3 small instances.

Our Contributions

Exploration of several directions to better solve BACP.

- Modelling : Different Balance Criteria

- Solving :
  - Branching Heuristic
  - Search Strategies
  - Dominance Rules

- Testing :
  - Instance Generator
  - Large Instances

Notations

The CP model is taken from Hnich et al, 2002.

- P_i is the period of course i (decision variables)
- w_i is the weight of course i
- $pred_i$ is the set of prerequisites of course i
- $succ_i$ is the set of courses for which i is a prerequisite
- L_j is the load of period j (secondary variables)

The Balance Criteria

Differant Criteria can be defined :

- Let L_i be the load of period i ($1 \leq i \leq m$) and w be the mean workload.
  - $C^{\max} = \max_{1 \leq i \leq m} L_i$ is the maximum load of a period.
  - $C(1) = \sum_{i=1}^{m} |L_i - w|$ is the sum of deviations from the mean load.
  - $C(2) = \sum_{i=1}^{m} (L_i - w)^2$ is the sum of square deviations from the mean load.
  - $C(\infty) = \max_{1 \leq i \leq m} |L_i - w|$ is the maximum deviation from the mean load.

- $C(2)$ is the “best” criterion :
  - $C(2)$ optimized $\Rightarrow$ near optimal other criteria.

Solving BACP in CP

Classic CP Approach : Branch & Bound with a min-domain/min-value branching heuristic.

Improvements :

- Branching Heuristic
- Search Strategies
- Dominance Rules

Branching Heuristic

- The <u>variable selection</u> follows a First-Fail heuristic.

  ☞ It selects the unassigned course with the smallest domain.

- The <u>value selection</u> tries to construct balanced solutions.

  ☞ It chooses the period that is the less heavily loaded.

- Size of the search tree is reduced in 93% of the instances with respect to a “min value” heuristic.

Search Strategies

- Branch-and-Bound
  - The balance is constrained to be less than some threshold.
  - Initially, this threshold is equal to a large upper bound.
  - Each time a solution is found, the threshold is decreased until the problem is unfeasible.
- Hybrid of Local Search + Branch-and-Bound
  - A Tabu Search yields a tight upper bound on the value of the balance.
  - Branch-and-Bound starts with this upper bound.
  - The Local Search Model is the same as the CP One.
- Iterative satisfaction problems
  - The balance is constrained to be less than some threshold.
  - Initially, this threshold is equal to some lower bound.
  - The threshold is incrementally increased until a solution is found.

Dominance Rules

- Dominance rules are a kind of unidirectional symmetries.
- Let s_i and s_j be states of the search tree
- Let $f(s)$ be a function that returns the value of the best solution that can be found from s.
- State s_i dominates s_j if $f(s_i) \leq f(s_j)$
- Useless to explore the subtree rooted in s_j once s_i is explored.
- We look for properties P such that constraining P will remove state dominated by others states.

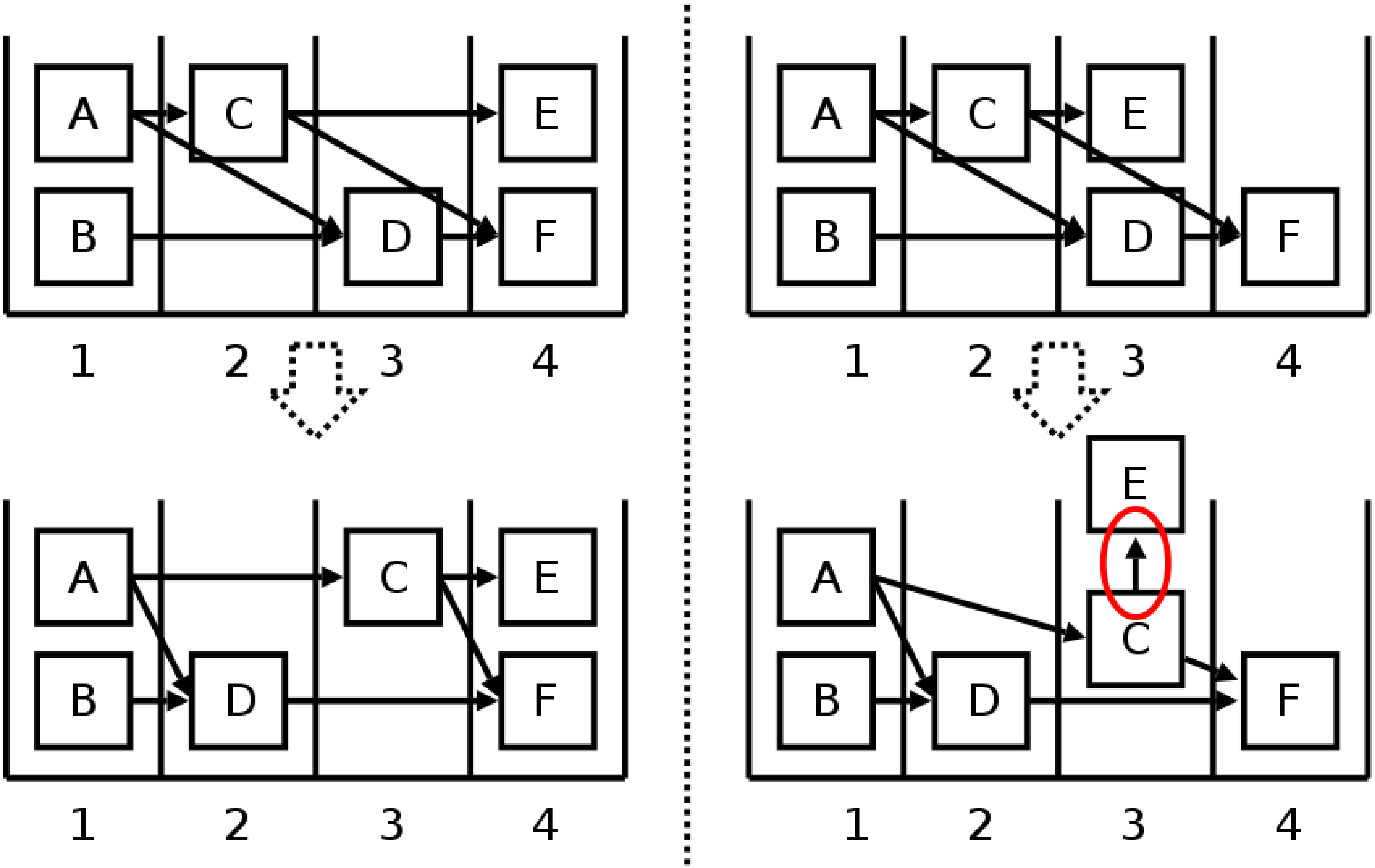
Dominance Rules in BACP

Posting a constraint $P_i \leq P_j$ preserves at least one optimally balanced solution to BACP if the following conditions hold :

- $w_i = w_j$
- $pred_i \subseteq pred_j$
- $succ_i \supseteq succ_j$

- The first condition ensures that swapping the periods of the two courses in a feasible assignment does not change the workload of the periods.
- The two others conditions make certain that whenever an optimal solution has the property that $P_i > P_j$, swapping the period of the two courses gives a feasible assignment (where $P_i \leq P_j$ holds).

Dominance Rules Examples



Dominance Rules Detection

- All dominance relations can be found in $\mathcal{O}(n^2.p)$ where n is the number of courses and p the number of prerequisites.

- For symmetric courses ($succ_i = succ_j$ and $pred_i = pred_j$), only one constraint can be posted.

- For 3 or more symmetric courses, cycles must be avoided (e.g. using a dichotomyc ordering).

Dominance Rules Application

Two possible approaches :

- Add all dominance rules at the root of the search tree.

  ☞ Can have a negative effect on the search heuristic as the first solution found might become inconsistent.

- Add rules dynamically during search (like SBDS):

  – Post the dominance constraint in the right branch of a binary search tree if this constraint holds in the (already explored) left branch.

  ☞ Removes states dominated by already explored states but not by states explored later during the search.

Experiments

- Instance generator to produce larger and harder instances.
- Test Set contains 720 instances with up to 200 courses and 40 periods.
- 3 Search Strategies :
  - BB : Branch and Bound
  - HY : Hybrid LS+BB
  - IT : Iterative Satisfaction
- 3 Dominance Applications :
  - - : None
  - D : Post Dominances at the root
  - DDS : Post Dominances During Search
- Experiment with the C^{max} and $C(1)$ criteria.

Results on the search strategies and the dominance rules

$C^{\max}$	-	D	DDS
BB	76.4	45.8	83.3
HY	96.5	94.0	95.7
IT	85.8	56.5	85.9

$C(1)$	-	D	DDS
BB	38.9	18.9	45.0
HY	76.5	70.4	76.9
IT	79.7	53.6	80.7

Table 1: Percentage of solved instances (on 720).

- The Hybrid search is the best for C^{max} and the Iterative approach for $C(1)$
- Adding dominances in the root (column “D”) degrades the search.
- Using dynamic dominances (column “DDS”) slightly improves the search.
- The Tabu Search reaches the optimal value for 93% and 77% of the instances, respectively with C^{max} and $C(1)$.

Conclusions

⋆ The BACP is a challenging problem and there is still to do.

⋆ State-of-the-Art limited to small instances.

⋆ Solving BACP with a large vision, exploring different directions to improve the resolution.

  – Modelling the Balance
  – Search Strategies
  – Dominance Rules
  – Branching Heuristic

⋆ Used Techniques can be applied to many other problems.

⋆ The definition, detection and application (SBDD-like) of dominance relations are open research areas.

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